Flux-Flow Viscosity Coefficient of Type-II Superconductors in the Mixed State

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(Received 29 September 1969)

The finite-temperature flux-flow viscosity coefficient of type-II superconductors in the mixed state is studied in the low-magnetic-field region. Both the effect of the penetration of the electric fields due to the surface charge distribution of the normal core of one flux line into the normal core regions of its neighboring flux lines and the effect of the thermal conduction are included. The latter is particularly important for dirty type-II superconductors. (1) It is shown that for dirty type-II superconductors in a low-magnetic field there exists a resistance minimum at a certain temperature $T_{\min}$, and that this temperature decreases with increasing magnetic field. This is in good agreement with the experimental data given by Kim, Hampstead, and Strnad. (2) The Hall angles for both the pure and the dirty type-II superconductors in the low-field region are shown to be consistent with the high-field experimental data, thus, explaining some of the mysterious features of the high-field data.

I. INTRODUCTION

The magnetic field penetrates into type-II superconductors in the mixed state in the form of quantized flux lines or vortices. The total magnetic flux of each quantized flux has a circulation corresponding to a single flux quantum $\varphi_0 = hc/2e$. When a transport electric current J, perpendicular to the magnetic field, passes through a type-II superconductor in the mixed state, the flux lines move in a steady state, determined by the force due to the transport current $\mathbf{J}\times\boldsymbol{\varphi}_0/c$, which balances the viscous drag $-\eta\mathbf{v}_L$. Here, $\boldsymbol{\varphi}_0$ is in the direction of the applied magnetic field, $\mathbf{v}_L$ is the velocity of a flux line, and η is the flux-flow viscosity coefficient, generally a function of the temperature of the system and the applied magnetic field $\eta = \eta(T)$.[1] The dissipation mechanisms associated with the moving flux lines, which originate the viscous drag, have been studied by many investigators.[2–4]

In a previous paper by the present author,[5] it is pointed out that the Bardeen-Stephen theory[3,4] is, in principle, valid only at zero temperature or in the zero-magnetic-field limit at finite temperatures. It is noticed that the original Bardeen-Stephen theory does not include the interaction between flux lines. It is further shown that if the penetration of the electric fields due to the surface charge distribution of the normal core of one flux line into the normal core regions of its neighboring flux lines is taken into account, the flux-flow viscosity coefficient should be modified to

$$\eta(T) = \eta(0)[1-\gamma H/H_{c2}(T)], \qquad (1)$$

in the low-magnetic-field region and at finite temperatures. Here, $\eta(0)$ denotes the zero-temperature flux-flow viscosity coefficient obtained by Bardeen and Stephen,

$$\eta(0) = \pi\hbar^2\sigma_n/2e^2a^2 = \varphi_0 H_{c2}(0)\sigma_n/c^2, \qquad (2)$$

where $H_{c2}(0)$ $[H_{c2}(T)]$ is the upper critical field of the type-II superconductor at zero temperature (at finite temperature T), σ_n is the normal metal conductivity at low temperatures, and a is the radius of a flux line, related to $H_{c2}(0)$ by $H_{c2}(0) = \hbar c/2ea^2$. The coefficient γ in Eq. (1) is a positive quantity depending weakly on the magnetic field and can be determined by comparing our theoretical conclusions with the experimental data. Based on Eq. (1), we obtained an expression for the flux-flow resistivity in the low-field region and at finite temperatures, which agrees qualitatively very well with the experimental data (for details, see Ref. 5). Yet, there are still a few unsettled problems with regard to the flux-flow resistivity of some high- (upper-critical-) field type-II superconducting alloys and the Hall angles of pure and dirty superconductors at finite temperatures. In the present paper, we shall look into these phenomena.

The relation between the zero-temperature flux-flow resistivity and the zero-temperature flux-flow viscosity coefficient is given by Bardeen and Stephen,

$$\rho_f(0) = \varphi_0 H/c^2\eta(0). \qquad (3)$$

It is natural that we can assume

$$\rho_f(T) = \varphi_0 H/c^2\eta(T) \qquad (4)$$

to be valid at finite temperatures, as the flux-flow viscosity coefficient is the only temperature-dependent quantity in Eq. (3). Kim *et al.*[1] observe that, in the low-field region, the flux-flow resistivity of some high- (upper-critical-) field type-II superconducting alloys peculiarly shows a minimum at a temperature somewhere between $0.3T_c$ and $0.4T_c$, and, further, the temperature at which the flux-flow resistivity minimum takes place $T_{\min}$ decreases with increasing magnetic field (see Figs. 7–9 of Ref. 1). With the flux-flow viscosity coefficient given by Eq. (1), we are not able to explain this phenomenon. Thus, a better expression for the flux-flow viscosity coefficient at finite temperatures is needed.

The Hall effect takes place in type-II superconductors in the mixed state because the presence of the magnetic

[1] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965).
[2] H. Suhl, Phys. Rev. Letters **14**, 226 (1965).
[3] M. J. Stephen and J. Bardeen, Phys. Rev. Letters **14**, 112 (1965).
[4] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).
[5] W. S. Chow, Phys. Rev. **188**, 783 (1969).

field in the core region of a flux line induces a component of flux-line motion in a direction parallel to the transport current **J**. Bardeen and Stephen find that, at $T=0$, the Hall angle $\alpha(0)$ is given by the following expression:

$$\tan\alpha(0)=(e\tau/mc)H\,, \tag{5}$$

where τ is the collision relaxation time of an electron in the normal metal. Making use of the relation $\sigma_n = Ne^2\tau/m$, where N is the density of electrons in the core region of the flux line and Eq. (2), we can rewrite Eq. (5) as

$$\tan\alpha(0)=(2/Nh)[H/H_{c2}(0)]\eta(0)\,. \tag{6}$$

Actual Hall-angle measurements are always carried out at finite temperatures. As in Eq. (6), the only quantity which is temperature-dependent is the flux-flow viscosity coefficient; again we can assume

$$\tan\alpha(T)=(2/Nh)[H/H_{c2}(0)]\eta(T) \tag{7}$$

to be valid at finite temperatures. Here, we do not change $H_{c2}(0)$ to $H_{c2}(T)$, since $H_{c2}(0)$ merely stands for a constant $\hbar c/2ea^2$. Again, we rely on a correct expression of the flux-flow viscosity coefficient to explain the Hall-angle experimental data. In the low-field region, the Hall angles are rather small. The Hall angle is determined through the measurement of the ratio of the voltage difference of a superconductor sample in the direction of the flux-line motion and that in the direction of the transport current. Experimentally, a small voltage difference in the direction of the flux-line motion is rather difficult to be measured accurately, and, for this reason, all Hall-angle experimental data reported in the literature are obtained in the high-field regions. In 1965, two sets of important high-field data were reported by two different groups. Reed, Fawcett, and Kim[6] of the Bell Telephone Laboratories reported that the measured Hall angles of a rather pure superconducting Nb are generally smaller than the values of the Bardeen-Stephen zero-temperature theory. The data were obtained at temperatures of 1.3 and 4.2°K. On the other hand, Niessen and Staas[7] of the Philips Research Laboratories reported that the measured Hall angles of a superconducting Nb-Ta alloy, which can be regarded as a dirty superconductor, are larger than the values given by the Bardeen-Stephen zero-temperature theory. For the dirty superconductor, the measured Hall angle increases from the value at $H=H_{c2}(T)$ with decreasing magnetic field until it reaches a maximum value; then it decreases with decreasing magnetic field. The data were obtained at temperatures 1.3, 2.0 and 4.3°K. The essential features of these two sets of the data are plotted in Fig. 1 (see also Fig. 9 of Ref. 8). It should be remarked

[6] W. A. Reed, E. Fawcett, and Y. B. Kim, Phys. Rev. Letters **14**, 790 (1965).

[7] A. K. Niessen and F. A. Staas, Phys. Letters **15**, 26 (1965).

[8] Y. B. Kim and M. J. Stephen, in *Superconductivity*, edited by R. D. Parks (M. Dekker, New York, 1969), Vol. II.

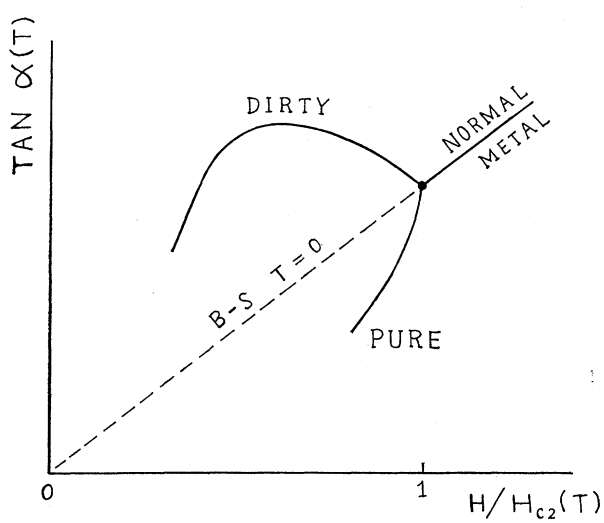

FIG. 1. The essential features of the high-field Hall-angle experimental data are plotted. *B-S* $T=0$ denotes the Bardeen-Stephen zero-temperature line.

that, up to now, there has not been a direct satisfactory high-field theory for these two sets of rather disconnected data. As a matter of fact, they provided quite a surprise to the superconductivity physicists in 1965. In the present paper, we merely intend to develop a low-field theory. As shall be shown, this theory is consistent with the high-field Hall-angle experimental data.

Now, the main problem is to obtain a better expression for the finite-temperature flux-flow viscosity coefficient in the low-field region.

II. FINITE-TEMPERATURE FLUX-FLOW VISCOSITY COEFFICIENT IN LOW-FIELD REGION

As shown in Ref. 5, if the penetration of the electric fields due to the surface charge distribution of the normal core of one flux line into the normal core regions of its neighboring flux lines is considered, the flux-flow viscosity coefficient should be modified to Eq. (1). Yet, this only represents part of the needed modification. As a matter of fact, in Eq. (1), we have neglected the contributions due to thermal conduction, which may be important at finite temperatures. Thus, the complete finite-temperature flux-flow viscosity coefficient should involve two terms; namely,

$$\eta(T)=\eta_E(T)+\eta_{\mathrm{Th}}(T)\,. \tag{8}$$

In the low-field region, $\eta_E(T)$ is given by Eq. (1):

$$\eta_E(T)=\eta(0)[1-\gamma H/H_{c2}(T)]\,. \tag{9}$$

The contribution due to thermal conduction $\eta_{\mathrm{Th}}(T)$ has been correctly found by Clem.[9] Following Bardeen and Stephen, Clem also considers an isolated moving

[9] J. R. Clem, Phys. Rev. Letters **20**, 735 (1968).

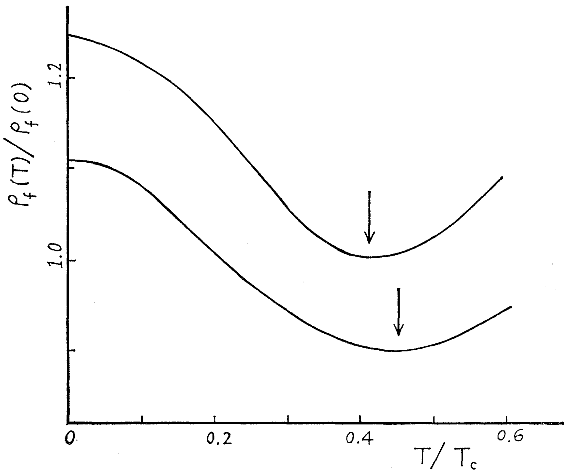

FIG. 2. $\rho_f(T)/\rho_f(0)$ is plotted against T/T_c. The lower curve corresponds to the magnetic field $\gamma H/H_{c2}(0)=0.1$ and the upper curve corresponds to $\gamma H/H_{c2}(0)=0.2$. The arrows indicate the temperatures at which the ratio $\rho_f(T)/\rho_f(0)$ takes the minimum values.

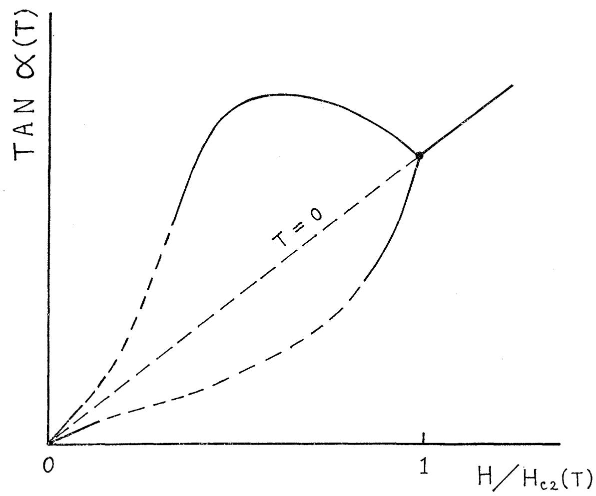

FIG. 3. The possible connections between the present low-field theory of the Hall angle and the high-field experimental data are shown.

flux line and notes that, to maintain the normal core (a region of higher entropy density at finite temperatures), to move at a constant velocity $\mathbf{v}_L$, the entropy currents must flow in such a way that they must supply entropy to the leading edge of the normal core and carry away entropy from the trailing edge of the normal core. Because of these entropy currents, a local temperature gradient is established inside and outside the moving normal-core region. Clem further calculates the energy dissipation associated with the moving flux line and obtains the thermal-conduction contributions to the flux-flow viscosity coefficient

$$\eta_{\mathrm{Th}}(T)=\pi a^2 T(S_n-S_s)^2/(K_n+K_s)\,, \tag{10}$$

where S_n (S_s) is the entropy density of normal (superconducting) electrons per unit volume and K_n (K_s) is the heat conductivity of the normal (superconducting) electrons. They are all functions of temperature. Following Clem, the ratio $\eta_{\mathrm{Th}}(T)/\eta(0)$ is expressed in terms of a product of a coefficient C and a function $F(T)$,

$$\eta_{\mathrm{Th}}(T)/\eta(0)=CF(T)\,, \tag{11}$$

where

$$\begin{aligned} C &\rightarrow 0 \quad \text{in the } \textit{pure} \text{ limit} \\ &\rightarrow 1 \quad \text{in the } \textit{dirty} \text{ limit}\,, \end{aligned} \tag{12}$$

and the function $F(T)$ is numerically given by Clem [see Fig. 1(b) of Ref. 9]. $F(T)$ is an increasing function of temperature until it reaches a maximum at $T=0.55T_c$, then it decreases until it reaches zero at $T=T_c$; with F $(0.55T_c)=0.29$ and $F(0)=F(T_c)=0$. Thus, it is expected that, for dirty superconductors, $\eta_{\mathrm{Th}}(T)$ will contribute significantly to $\eta(T)$ in the temperature region $0.2T_c<T<0.9T_c$. The behavior of $F(T)$ as a function of temperature can also be qualitatively observed in Eq. (10).

It should be pointed out that Clem's result, Eq. (11), is again only valid in the low-field region, since the influence of the temperature gradient outside the core region of one flux line on the entropy currents inside the core regions of its neighboring flux lines is not considered. This influence is certainly important in the higher field regions.

Now, with the thermal-conduction contributions included, we have, for *pure* type-II superconductors,

$$\eta(T)\cong\eta(0)[1-\gamma H/H_{c2}(T)] \tag{13}$$

and for *dirty* type-II superconductors, or type-II superconducting alloys,

$$\eta(T)\cong\eta(0)[1-\gamma H/H_{c2}(T)+F(T)]. \tag{14}$$

III. FLUX-FLOW RESISTANCE MINIMUM IN HIGH- (UPPER-CRITICAL-) FIELD TYPE-II SUPERCONDUCTING ALLOYS

In this section, we shall explain qualitatively the low-field resistance data obtained by Kim *et al.* (Figs. 7–9 of Ref. 1) for the high-field type-II superconducting alloy. The data are measured in the temperature region from 1 to 4.2°K, corresponding to upper critical field $H_{c2}(T)$ ranging from 35 to 25 kG. The transition temperature of the sample is $\sim$9°K. The magnetic fields under which the resistance measurements are performed are 4, 7, and 10 kG. It is doubtful whether 10 kG can be regarded as low field in our theory. But the 4 and 7 kG data are sufficient to indicate the general behavior of the field dependence of the temperature $T_{\min}$ at which the resistance minimum takes place. The measured ratio R/R_n, which is equal to $\rho_f(T)/\rho_n$ with $\rho_n=\sigma_n^{-1}$, shows a minimum in the vicinity of $T\sim3$°K.

Substituting Eq. (14) into Eq. (4), we obtain

$$\rho_f(T)/\rho_n = H/H_{c2}(0)[1-\gamma H/H_{c2}(T)+F(T)]^{-1}. \quad (15)$$

As $\rho_f(0)/\rho_n = H/H_{c2}(0)$, we can rewrite the above equations as

$$\rho_f(T)/\rho_f(0) = [1-\gamma H/H_{c2}(T)+F(T)]^{-1}. \quad (16)$$

In Fig. 2, we plot $\rho_f(T)/\rho_f(0)$ against the reduced temperature T/T_c, with $\gamma H/H_{c2}(0)$ the parameter. The values of $H_{c2}(T)$ are taken from Fig. 9 of Ref. 1. The two curves shown in the figure correspond to the parameter $\gamma H/H_{c2}(0)=0.1$ and 0.2. We notice that the values of $\rho_f(T)/\rho_f(0)$ are generally larger for larger value of $\gamma H/H_{c2}(0)$ or larger magnetic field H. The temperature $T_{\min}$ at which the resistance minimum takes place does decrease with increasing magnetic field H. The results are

(a) $T_{\min}\cong 0.45T_c$ or 4.1°K when $\gamma H/H_{c2}(0)=0.1$,

(b) $T_{\min}\cong 0.41T_c$ or 3.7°K when $\gamma H/H_{c2}(0)=0.2$.

With $H_{c2}(0)=36$ kG, if we choose $\gamma=1$, the two corresponding magnetic fields will be: (a) $H=3.6$ kG and (b) $H=7.2$ kG. The numerical results shown here do explain qualitatively the experimental data in Fig. 8 of Ref. 1. Further, our results also agree very well with the independent Pb-Tl alloy experimental data of Axt and Joiner.[10]

Clem[9] fails to show the field dependence of the temperature at which the resistance minimum takes place because he has used an inadequate expression for $\eta_E(T)$, namely, $\eta_E(T)/\eta(0)=H_{c2}(T)/H_{c2}(0)$. This expression is inadequate because with Eq. (4) it leads to an inadequate relation $\rho_f(T)/\rho_n=H/H_{c2}(T)$ which has been criticized in Ref. 5.

IV. LOW-FIELD HALL ANGLES AT FINITE TEMPERATURES

In this section, we shall show that based on our expressions, Eqs. (7), (13), and (14), we can obtain Hall angles in the low-field region.

A. Pure Type-II Superconductors

In the pure limit, the low-field Hall angle is given by

$$\tan\alpha(T) = \frac{2}{Nh}\eta(0)\frac{H}{H_{c2}(0)}\left(1-\frac{\gamma H}{H_{c2}(T)}\right) \xrightarrow[H\to 0]{} \frac{2}{Nh}\eta(0)\frac{H}{H_{c2}(T)}\left\{\frac{H_{c2}(T)}{H_{c2}(0)}\right\}. \quad (17)$$

[10] C. J. Axt and W. C. H. Joiner, Phys. Rev. Letters 21, 1168 (1968).

The quantity in the curly bracket is always less than 1 at finite temperatures. Thus, in a $\tan\alpha(T)$-versus-$H/H_{c2}(T)$ plot, we expect that $\tan\alpha(T)$ will start at $H/H_{c2}(T)=0$ with a slope smaller than that of the Bardeen-Stephen zero-temperature line. This connects very well with the high-field experimental data on pure superconductors.

B. Dirty Type-II Superconductors

In the dirty limit, we have

$$\tan\alpha(T) = \frac{2}{Nh}\eta(0)\frac{H}{H_{c2}(0)}\left(1-\frac{\gamma H}{H_{c2}(T)}+F(T)\right) \xrightarrow[H\to 0]{} \frac{2}{Nh}\eta(0)\frac{H}{H_{c2}(T)}\left\{\frac{H_{c2}(T)}{H_{c2}(0)}[1+F(T)]\right\}. \quad (18)$$

The quantity in the curly bracket now may be larger than 1 in the low-temperature region, say $T<0.5T_c$, as in this temperature region $H_{c2}(T)/H_{c2}(0)\gtrsim 1$, and we also take into account the fact that the function $F(T)$ as given by Clem involves various approximations. If the quantity in the curly bracket is larger than 1, then in a $\tan\alpha(T)$-versus-$H/H_{c2}(T)$ plot, we can have $\tan\alpha(T)$ to start at $H/H_{c2}(T)=0$ with a slope larger than that of the Bardeen-Stephen zero-temperature line. This would connect well with the high-field dirty superconductor experimental data.

In Fig. 3, we show the possible connections between the low-field theory and the high-field experimental data. Although this part of the discussion is rather speculative, we do succeed in explaining some of the mysterious features of the high-field Hall angle experimental data.

V. CONCLUSION

In conclusion, we point out that the present theory of the finite-temperature flux-flow viscosity coefficient of type-II superconductors in the mixed state is rather qualitative and phenomenological. This is so because the present theory is based on a rather crude local model of the flux line in the Bardeen-Stephen theory. Yet, we do succeed in explaining qualitatively the behavior of the low-field flux-flow resistance minimum in high-field type-II superconducting alloys and in lifting some of the mysterious features of the Hall-angle experimental data in the high-field region. The future improvements of the theory necessarily rely on our better understanding of the more realistic nonlocal model of the flux line, as suggested by Bardeen and Stephen.